Optimal prevention and adaptation in a stochastic flood model

An application of vintage models in socio-hydrology

Michael Freiberger
Supervisor: Alexia Fürnkranz-Prskawetz

Institute of Statistics and Mathematical Methods in Economics
Vienna Doctoral Programme on Water Resource Systems

September, 18^{th} 2018

Vienna Doctoral Programme on Water Resource Systems
www.waterresources.at

Der Wissenschaftsfonds.

TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

Overview

1. What is socio-hydrology?
2. Model introduction stochastic problem formulation
3. Transformation (into a vintage model)
4. Benchmark solution
5. Sensitivity analysis

What is socio-hydrology?

Socio-hydrology aims to describe the two-way coupled feedbacks between water systems and human behaviour:

- The water system impacts the behaviour of the population. (e.g. flood risk prevention)
- Human behaviour also affects the water system. (e.g. building of dikes)

Until recently research mainly focused on quantitative model analysis.

Stochastic flood events and optimal behaviour

Two different present approaches for flood models:

- Stochastic modelling of high water levels with a priori defined economic reactions and behaviour (e.g. Viglione et al. 2014)
- Modelling the social optimal decisions with deterministic appearance of high water levels (e.g. Grames et al. 2016)

Stochastic flood events and optimal behaviour

Two different present approaches for flood models:

- Stochastic modelling of high water levels with a priori defined economic reactions and behaviour (e.g. Viglione et al. 2014)
- Modelling the social optimal decisions with deterministic appearance of high water levels (e.g. Grames et al. 2016)

My aim: Combine the two approaches and derive social optimal behaviour under stochastic appearance of high water levels.
This includes two parts

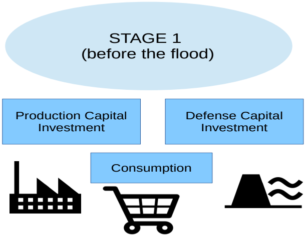
- Optimal decisions before a flood occurs (prevention)
- Optimal reaction after flood occurrence (adaptation)

Overview

1. What is socio-hydrology?
2. **Model introduction and stochastic problem formulation**
3. Transformation (into a vintage model)
4. Benchmark solution
5. Sensitivity analysis

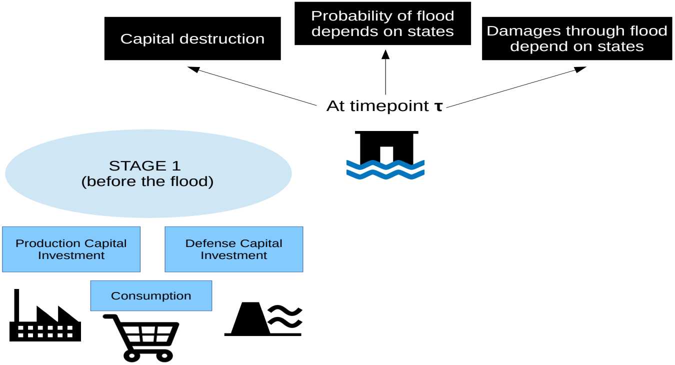
Basic model structure

We formulate a 2-stage optimal control problem with stochastic switching time:



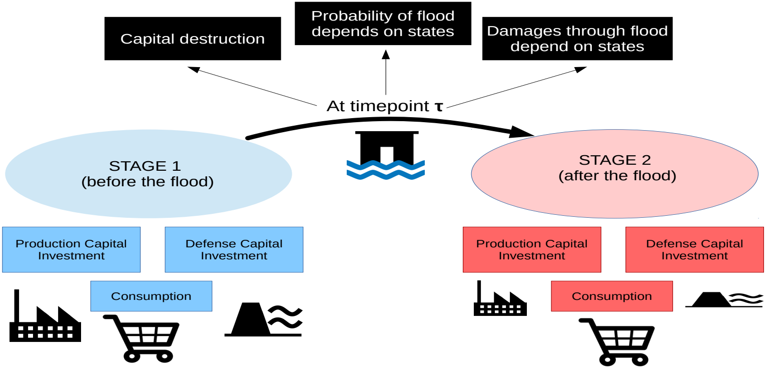
Basic model structure

We formulate a 2-stage optimal control problem with stochastic switching time:



Basic model structure

We formulate a 2-stage optimal control problem with stochastic switching time:



Stochastic Model formulation

First stage optimisation

$$\max_{c(t),i_d(t)} \mathbb{E}_\tau \left[\int_0^\tau e^{-\rho t} u(c(t))dt + e^{-\rho\tau} V^*(K_i(\tau),\tau) \right]$$

s.t.: $\dot{K}_y(t) = AK_y(t)^\alpha - c(t) - Q(i_d(t)) - \delta_y K_y(t)$ $\qquad K_y(0) = K_y^0$

$\dot{K}_d(t) = i_d(t) - \delta_d K_d(t)$ $\qquad K_d(0) = K_d^0$

Stochastic Model formulation

First stage optimisation

$$\max_{c(t),i_d(t)} \mathbb{E}_\tau \left[\int_0^\tau e^{-\rho t} u(c(t))dt + e^{-\rho\tau} V^*(K_i(\tau),\tau) \right]$$

$$\text{s.t.:} \quad \dot{K}_y(t) = AK_y(t)^\alpha - c(t) - Q(i_d(t)) - \delta_y K_y(t) \qquad K_y(0) = K_y^0$$

$$\dot{K}_d(t) = i_d(t) - \delta_d K_d(t) \qquad K_d(0) = K_d^0$$

Second stage optimisation

$$V^*(K_i(\tau),\tau) = \max_{c(t),i_d(t)} \int_\tau^T e^{-\rho(t-\tau)} u(c(t))dt + e^{-\rho(T-\tau)} SalvageValue$$

$$\text{s.t.:} \quad \dot{K}_y(t) = AK_y(t)^\alpha - c(t) - Q(i_d(t)) - \delta_y K_y(t) \qquad K_y(\tau) = \lim_{s\to\tau^-} (1 - d(K_d(s)))K_y(s)$$

$$\dot{K}_d(t) = i_d(t) - \delta_d K_d(t) \qquad K_d(\tau) = \lim_{s\to\tau^-} (1 - d(K_d(s)))K_d(s)$$

Properties of the switching time τ

The switching time = occurrence of a flood, which depends on two factors:

- Exogenous given stochastic appearances of high water levels. E.g. Poisson-distributed arrival $\longrightarrow$ exponentially distributed time until flood.

Properties of the switching time τ

The switching time = occurrence of a flood, which depends on two factors:

- Exogenous given stochastic appearances of high water levels. E.g. Poisson-distributed arrival $\longrightarrow$ exponentially distributed time until flood.
- Probability that the high water level surpasses the size of the defense capital (i.e. dikes).

Properties of the switching time τ

The switching time = occurrence of a flood, which depends on two factors:

- Exogenous given stochastic appearances of high water levels. E.g. Poisson-distributed arrival $\longrightarrow$ exponentially distributed time until flood.
- Probability that the high water level surpasses the size of the defense capital (i.e. dikes).

We can derive the density function of flood occurrence as the product of the two terms.

Properties of the damage function

Furthermore also the damage (and therefore the initial values for the second stage) depend on the defense capital and the high water level.

$$d(K_d(t), W(t)) = \begin{cases} 1 - e^{-(W(t)+\xi_d K_d(t))} & \text{if} \quad W(t) + \xi_d K_d(t) \geq K_d(t) \\ 0 & \text{else} \end{cases}$$

Properties of the damage function

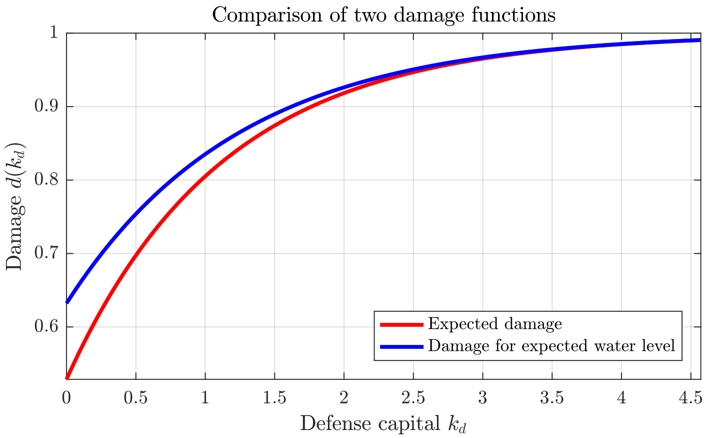
Furthermore also the damage (and therefore the initial values for the second stage) depend on the defense capital and the high water level.

$$d(K_d(t), W(t)) = \begin{cases} 1 - e^{-(W(t)+\xi_d K_d(t))} & \text{if} \quad W(t) + \xi_d K_d(t) \geq K_d(t) \\ 0 & \text{else} \end{cases}$$

We can define the damage (=share of capital which gets destroyed) for the model as the damage from the expected water level or the expected damage, i.e.

$$d(K_d(t)) := d(K_d(t), \mathbb{E}W(t)) \qquad \text{or} \qquad d(K_d(t)) := \mathbb{E}d(K_d(t), W(t))$$

Properties of the damage function



Overview

Transformation into a vintage model — Boukas (1990) + Wrzaczek (2017)

We introduce every possible switching time as a vintage s in the model additionally:

$$\text{Before the flood} \longrightarrow \text{After the flood at timepoint } s$$

$$K_i(t) \longrightarrow \widetilde{K}_i(t,s)$$

$$c(t) \longrightarrow \widetilde{c}(t,s)$$

$$i_d(t) \longrightarrow \widetilde{i_d}(t,s)$$

Transformation into a vintage model — Boukas (1990) + Wrzaczek (2017)

We introduce every possible switching time as a vintage s in the model additionally:

$$\text{Before the flood} \longrightarrow \text{After the flood at timepoint } s$$

$$K_i(t) \longrightarrow \widetilde{K}_i(t,s)$$

$$c(t) \longrightarrow \widetilde{c}(t,s)$$

$$i_d(t) \longrightarrow \widetilde{i}_d(t,s)$$

What are the benefits of this transformation?

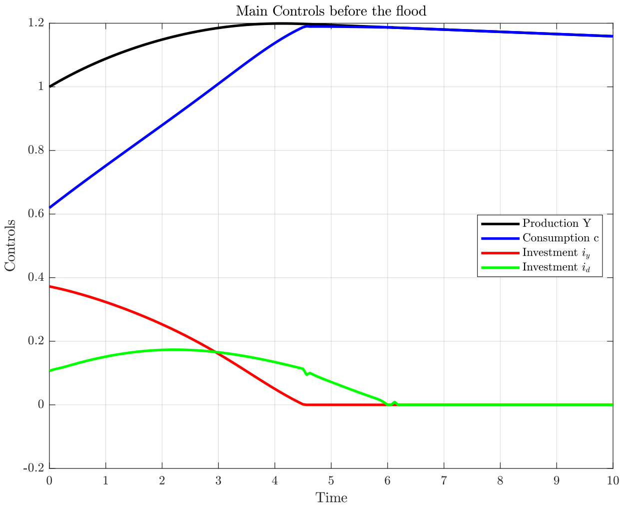
- Can be solved more efficiently in numerical analysis.
- Optimal decisions for all (temporal) possible flood scenarios.

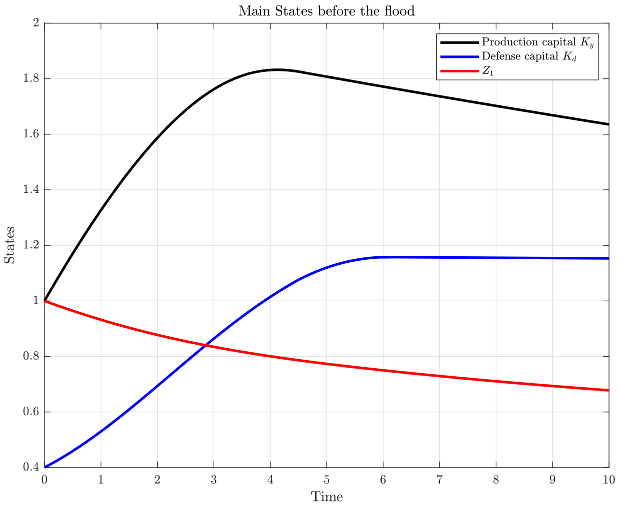
Overview

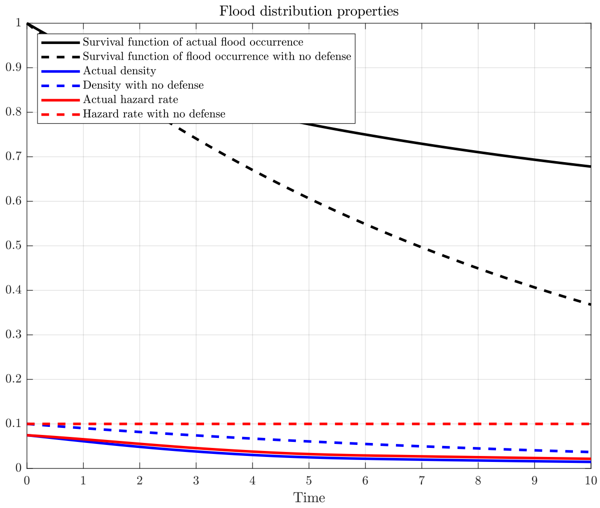
Benchmark-Model

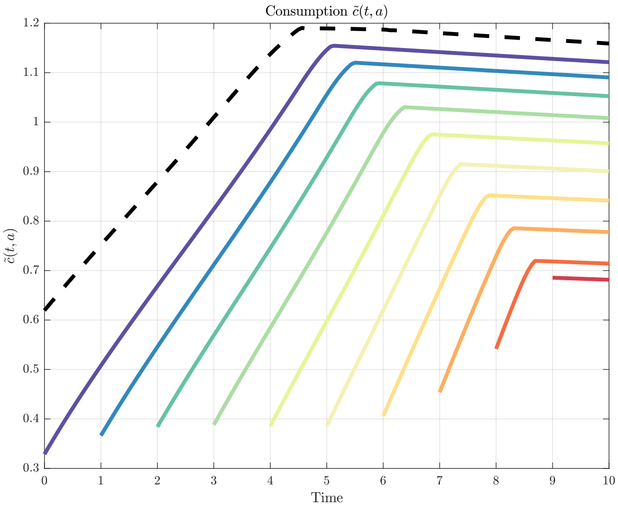
Time horizont	T	10
Utility/Preferences	ρ	0.03
Depreciation rates	δ_y	0.02
	δ_d	0.001
Flood damages	ξ_d	0.1
Stochastic properties	θ_3	0.28
of floods	λ	0.1
Initial capital	K_y^0	1
stocks	K_d^0	0.4
Salvage values	$S_y(K_y)$	0
	$S_d(K_d)$	0

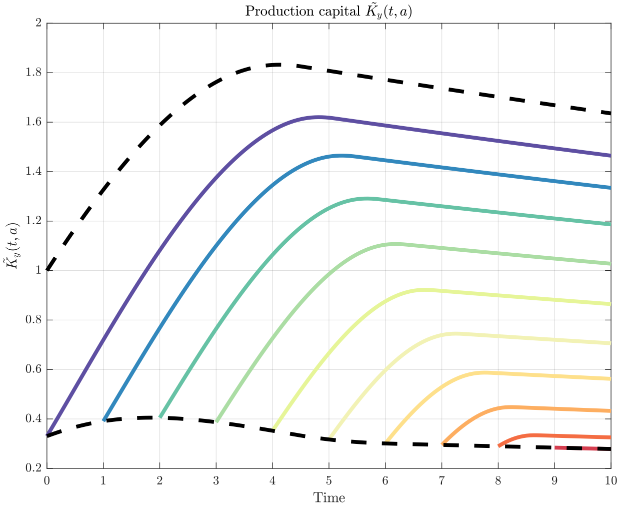
Table: Parameter values for the benchmark solution


Main Controls before the flood
Controls
Time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
5
6
7
8
9
10
Production Y
Consumption c
Investment i_y
Investment i_d


Main States before the flood
States
Time
Production capital K_y
Defense capital K_d
Z_1
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0
1
2
3
4
5
6
7
8
9
10


Flood distribution properties
Survival function of actual flood occurrence
Survival function of flood occurrence with no defense
Actual density
Density with no defense
Actual hazard rate
Hazard rate with no defense
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 1 2 3 4 5 6 7 8 9 10
Time


Consumption $\tilde{c}(t,a)$
$\tilde{c}(t,a)$
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
1
2
3
4
5
6
7
8
9
10
Time


Production capital $\tilde{K}_y(t,a)$
$\tilde{K}_y(t,a)$
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
Time

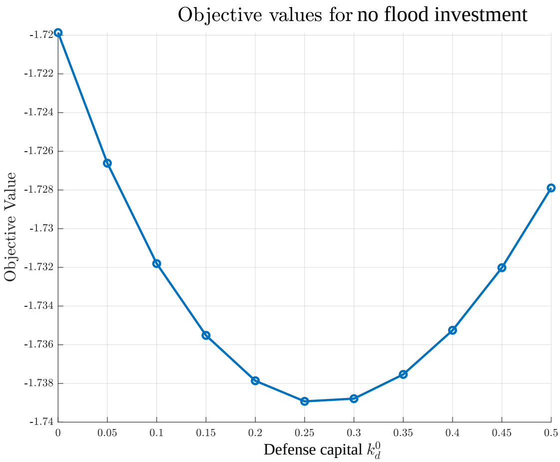
Overview

1. What is socio-hydrology?
2. Model introduction and stochastic problem formulation
3. Transformation (into a vintage model)
4. Benchmark solution
5. **Sensitivity analysis**

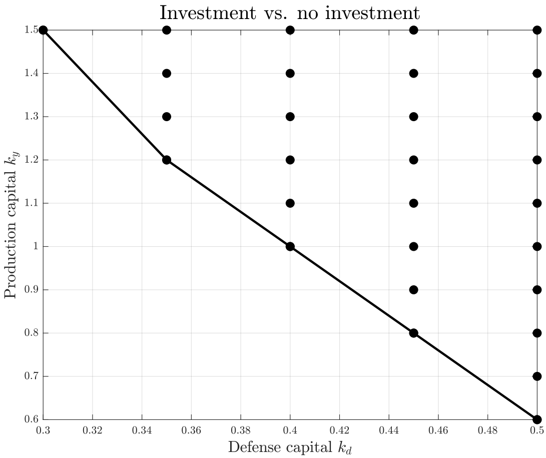
Benefits of flood prevention (Sensitivity analysis)

- Analyse the optimal behaviour with and without possible investment in defense capital (i.e. $i_d(t) = 0$)
- Sensitivity analysis w.r.t.
  - the initial level of defense capital k_d,
  - the initial level of production capital k_y,
  - the time horizon T,
  - the cost structure of flood protection investments θ_1.

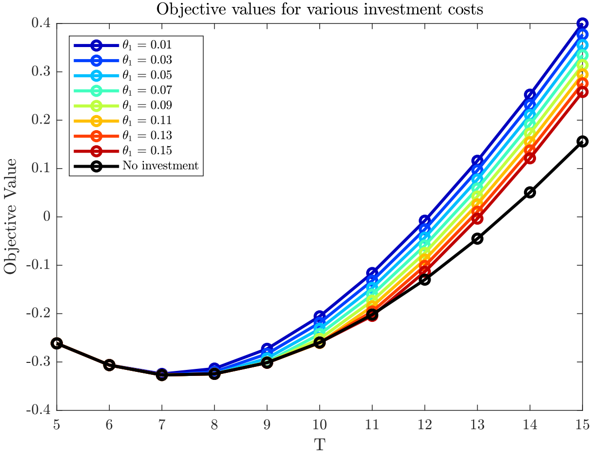
Initial capital stocks



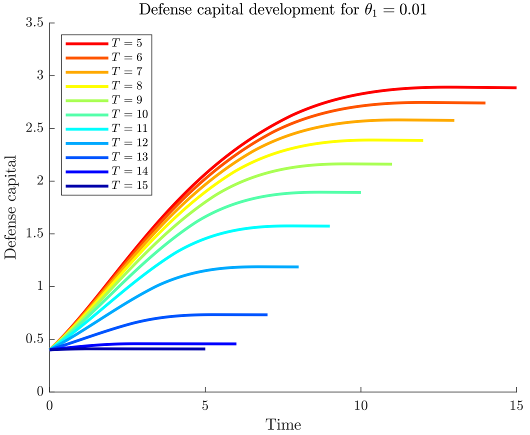
Initial capital stocks



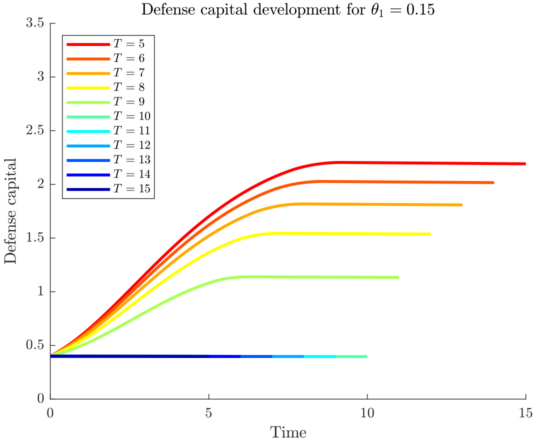
Time horizon and prevention costs



Time horizon and prevention costs



Time horizon and prevention costs



Optimal prevention and adaptation in a stochastic flood model

An application of vintage models in socio-hydrology

Michael Freiberger
Supervisor: Alexia Fürnkranz-Prskawetz

Institute of Statistics and Mathematical Methods in Economics
Vienna Doctoral Programme on Water Resource Systems

September, 18th 2018

Vienna Doctoral Programme on Water Resource Systems
www.waterresources.at

FWF Der Wissenschaftsfonds.

TECHNISCHE UNIVERSITÄT WIEN Vienna University of Technology

Additional material

$$\max_{c(t),i_d(t)} \mathbb{E}_\tau \left[\int_0^\tau e^{-\rho t} U(c(t))dt + e^{-\rho\tau} V^*(x(\tau),\tau) \right]$$

$$\text{s.t.:} \quad \dot{k}_y(t) = Ak_y(t)^\alpha - c(t) - Q(i_d(t)) - \delta_y k_y(t) \qquad k_y(0) = k_0^y$$

$$\dot{k}_d(t) = i_d(t) - \delta_d k_d(t) \qquad k_d(0) = k_0^d$$

$$V^*(x(\tau),\tau) = \max_{c(t),i_d(t)} \int_\tau^T e^{-\rho(t-\tau)} U(c(t))dt + e^{-\rho(T-\tau)} (S(k_y(T)) + S(k_d(t)))$$

$$\text{s.t.:} \quad \dot{k}_y(t) = Ak_y(t)^\alpha - c(t) - Q(i_d(t)) - \delta_y k_y(t)$$

$$k_y(\tau) = \lim_{s\to\tau^-} (1 - d(k_d(s)))k_y(s)$$

$$\dot{k}_d(t) = i_d(t) - \delta_d k_d(t)$$

$$k_d(\tau) = \lim_{s\to\tau^-} (1 - d(k_d(s)))k_d(s)$$

High water level / flood distribution

We can show that the following property holds (conditional distribution function)

$$\mathbb{P}\Big[W(t) \leq w | W(t) \geq k_d(t) - \xi_d k_d(t)\Big] = 1 - \left(\frac{1+\theta_3-\theta_3 w}{1+\theta_3-\theta_3(1-\xi_d)k_d(t)}\right)^{\frac{1}{\theta_3}}$$

After tedious calculations we can show that

$$\mathbb{E}HW(t) = 1 + \frac{(1-\xi_d)}{1+\theta_3} k_d(t)$$

And therefore the damage function

$$d(k_d(t)) := d(k_d(t), \mathbb{E}HW(t)) = 1 - e^{-\left(1+\frac{1+\theta_3\xi_d}{1+\theta_3}k_d(t)\right)}$$

Expected damage

As $d(...)$ is concave in $HW(t)$ it holds

$$\mathbb{E}d(k_d(t), HW(t)) < d(k_d(t), \mathbb{E}HW(t))$$

- Therefore using the expected damage might be more accurate.
- But the expected damage is far more complicated and can not be done analytically for all values of θ_3. (Incomplete Gamma function or complicated summations)
- Nevertheless we will further assume we have a damage function $d(k_d(t))$.

Distribution of flood occurrence

Assume distribution similar to Viglione (2014)

- The arrival time of high water levels is exponentially distributed

$$f_{exp}(t) = \lambda e^{-\lambda t} \cdot \mathbb{1}_{[0,\infty)}(t)$$

  with the mean value of $\frac{1}{\lambda}$

- The high-water level (at occurrence) is Pareto-distributed.

$$\mathbb{P}\Big[W \leq w | \Upsilon(t) = 0\Big] = 1 - \left(1 - \frac{\theta_3}{1+\theta_3} w\right)^{1/\theta_3}$$

Need probability, that high water level leads to flood

$$p(t) = \mathbb{P}\Big[W(t) + \xi_d k_d(t) > k_d(t)\Big] = 1 - \mathbb{P}\Big[W(t) \leq (1-\xi_d)k_d(t)\Big]$$

$$= \left(1 - \frac{\theta_3}{1+\theta_3}(1-\xi_d)k_d\right)^{1/\theta_3}$$

Now we can calculate the distribution function of the time of flood appearance τ by

$$\mathbb{P}\left[\tau \leq t\right] = \int_0^t f_{exp}(\tilde{t})p(\tilde{t})d\tilde{t} = \int_0^t f_{fl}(\tilde{t})d\tilde{t} =: F_{fl}(t)$$

and we obtain the hazard rate

$$\eta_{fl}(t) = \frac{f_{exp}(t)p(t)}{1 - \int_0^t f_{exp}(\tilde{t})p(\tilde{t})d\tilde{t}} = \frac{f_{exp}(t)p(t)}{1 - F_{fl}(t)}$$

The damage function

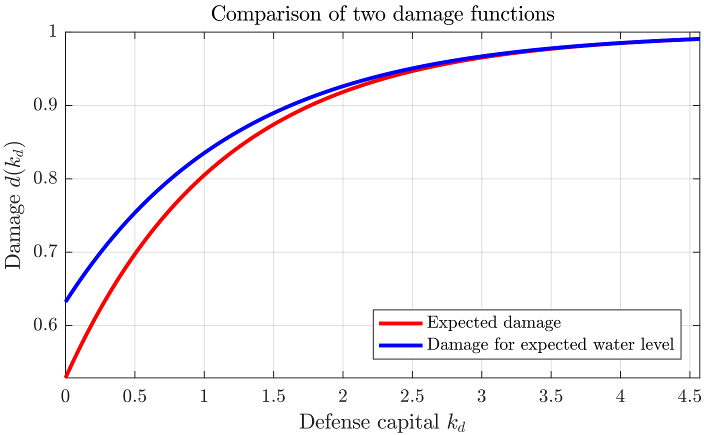
Problem that damage is still stochastic as water level above dike height is stochastic.

$$d(k_d(t), W(t)) = \begin{cases} 1 - e^{-(W(t)+\xi_d k_d(t))} & \text{if} \quad W(t) + \xi_d k_d(t) \geq k_d(t) \\ 0 & \text{else} \end{cases}$$

Using the expected water level leads to

$$d(k_d(t)) := d(k_d(t), \mathbb{E}HW(t)) = 1 - e^{-\left(1 + \frac{1+\theta_3 \xi_d}{1+\theta_3} k_d(t)\right)}$$

Properties of the damage function



Transformation into a det. model— Boukas (1990)

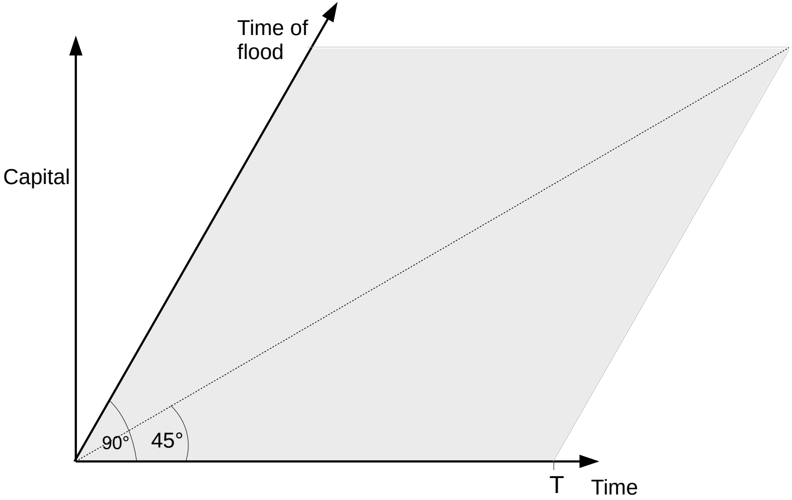
$$\max_{c(t), i_d(t)} \int_0^T e^{-\rho t} Z_1(t) \left[u(c(t)) + \eta_{fl}\Big(t, K_d(t), Z_1(t)\Big) V^*(x(t), t) \right] dt$$

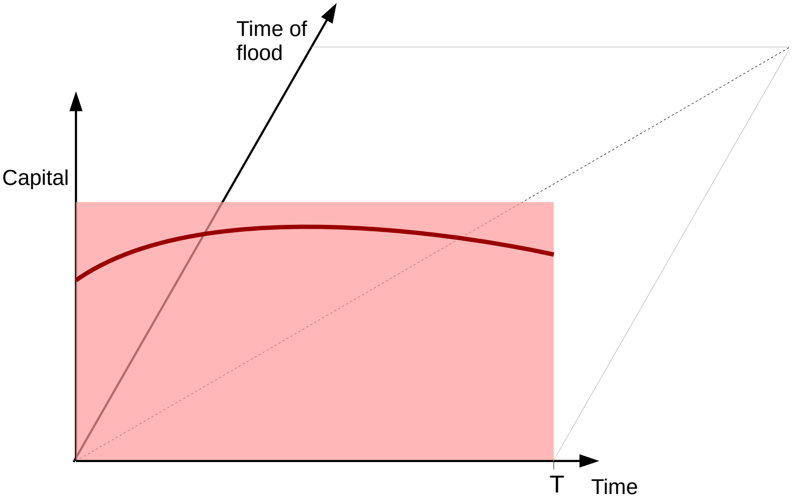
$$\text{s.t.:} \quad \dot{K}_y(t) = AK_y(t)^{\alpha} - c(t) - Q(i_d(t)) - \delta_y K_y(t) \quad K_y(0) = K_0^y$$

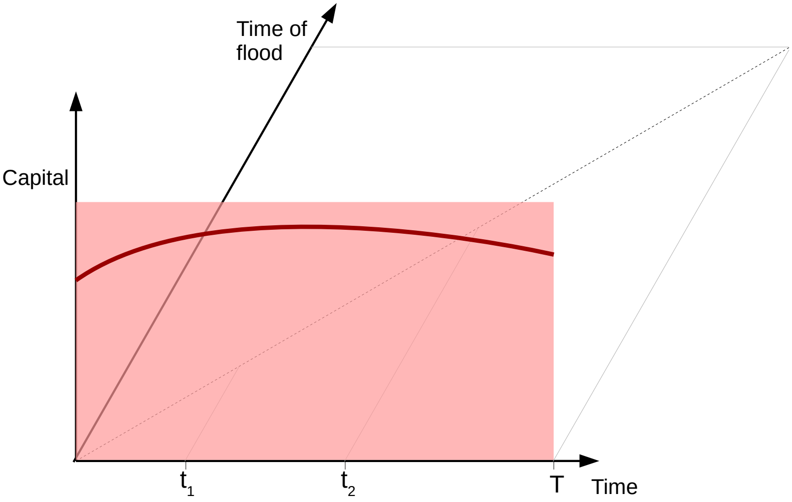
$$\dot{K}_d(t) = i_d(t) - \delta_d K_d(t) \quad K_d(0) = K_0^d$$

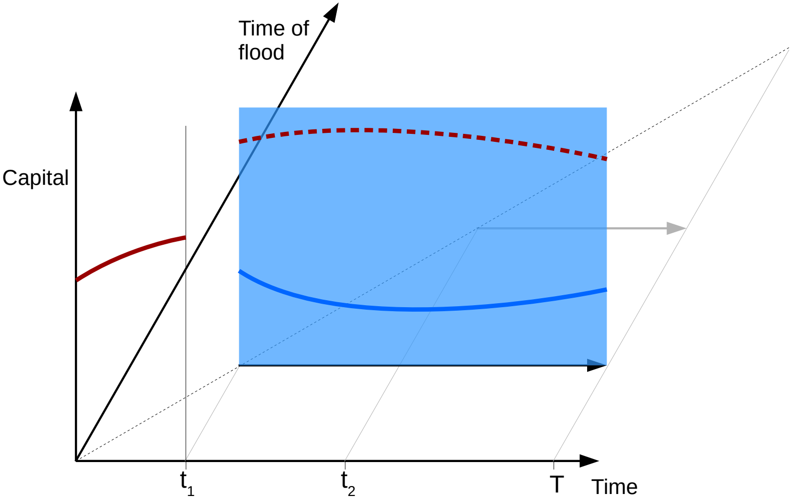
$$\dot{Z}_1(t) = -f_{exp}(t) p(K_d(t)) \quad Z_1(0) = 1$$

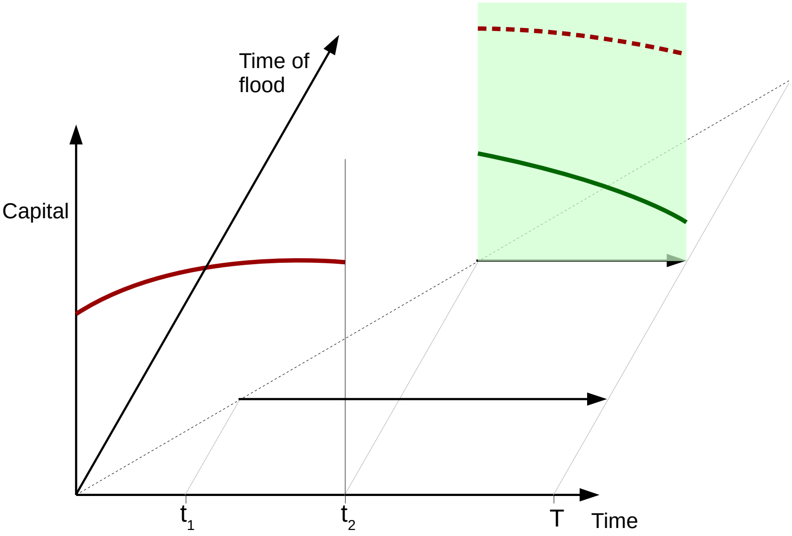
with the same second stage as before.

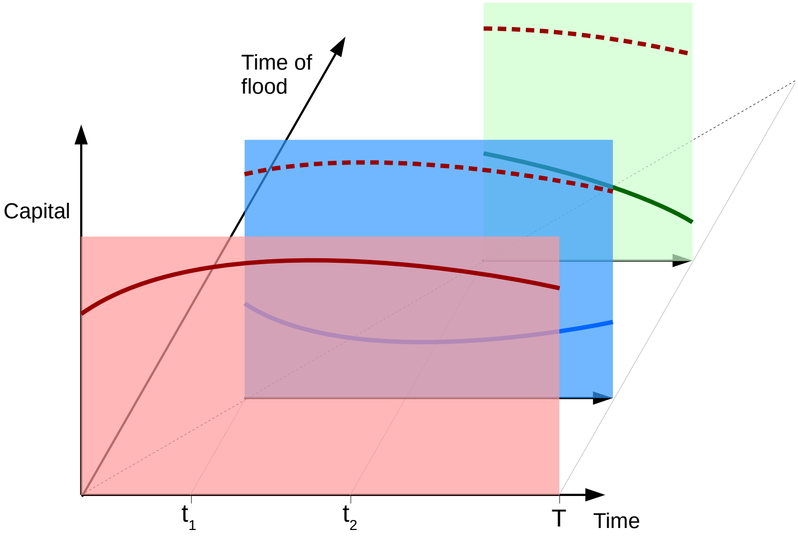

Time of
flood
Capital
90°
45°
T
Time


Time of
flood
Capital
T
Time


Time of
flood
Capital
t1
t2
T
Time


Time of
flood
Capital
t1
t2
T
Time


Time of
flood
Capital
t1
t2
T
Time


Time of
flood
Capital
t1
t2
T
Time

Properties of two stage optimal control problems

Deterministic two-stage optimal control problems can in general be solved numerically, but have several disadvantages:

Properties of two stage optimal control problems

Deterministic two-stage optimal control problems can in general be solved numerically, but have several disadvantages:

- The objective value of the second stage has to be theoretically calculated for every possible starting point and all possible initial state combinations.

Properties of two stage optimal control problems

Deterministic two-stage optimal control problems can in general be solved numerically, but have several disadvantages:

- The objective value of the second stage has to be theoretically calculated for every possible starting point and all possible initial state combinations.
- This is either numerically very expensive or the objective function has to be interpolated.

Properties of two stage optimal control problems

Deterministic two-stage optimal control problems can in general be solved numerically, but have several disadvantages:

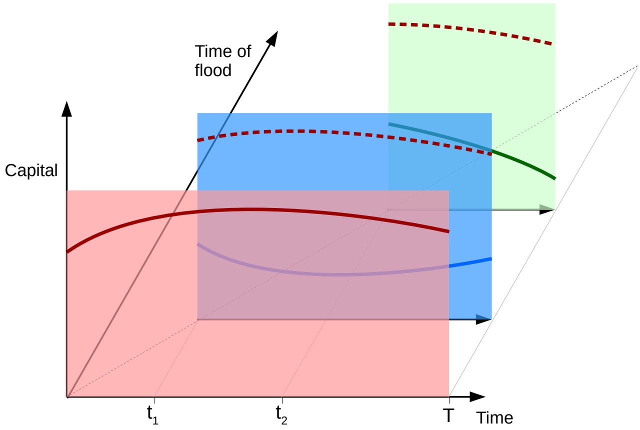
- The objective value of the second stage has to be theoretically calculated for every possible starting point and all possible initial state combinations.
- This is either numerically very expensive or the objective function has to be interpolated.
- If the objective function gets interpolated we loose a lot of information about the optimal controls in the second stage.

Difference between stochastic and vintage formulation

Stochastic formulation:

- Analyse solution along each panel.
- Aggregate over all panels

Difference between stochastic and vintage formulation



Difference between stochastic and vintage formulation

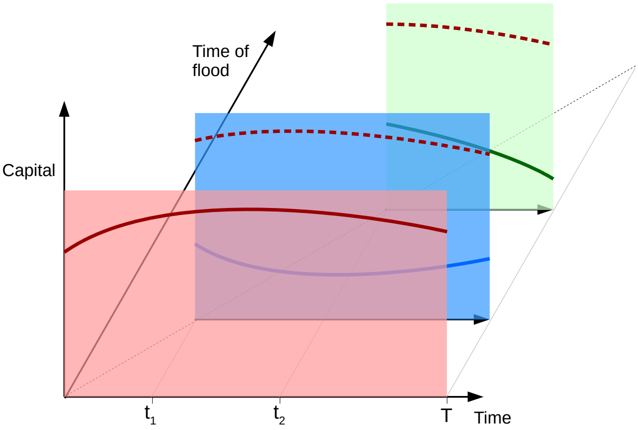
Stochastic formulation:

- Analyse solution along each panel.
- Aggregate over all panels

Vintage formulation

- Analyse cross-section of all panels at one time-point
- Aggregate over all time-points

Difference between stochastic and vintage formulation



Difference between stochastic and vintage formulation

Stochastic formulation:

- Analyse solution along each panel.
- Aggregate over all panels

Vintage formulation

- Analyse cross-section of all panels at one time-point
- Aggregate over all time-points

Vintage formulation now has huge advantages:

- Existing algorithms for vintage-model can be adapted to obtain numerical solutions.
- Results deliver optimal behaviour for every possible flood scenario.
- Information about the optimal solution across panels can be obtained.