Training on concepts and terminology for analytics related to disaster risk finance and insurance

Block 2: Risk Metrics & Monte Carlo Simulation. A country case study.

José Ángel Villalobos, Darío Bacchini and Barry Maher
Block 2: Cat modelling and Financial Risk Metrics

Agenda

10:45 – 12:00
• Why to quantify, before occurrence, natural disaster financial losses?
• Early Natural Catastrophe Risk analysis applied to Insurance
• Loss distributions and simulation

13:00 – 13:45
• Risk Metrics, then and now
• Using Risk metrics
December 23rd, 1972: Nicaragua, Managua EQ

- 6.9 Richter Scale, just 5 Km. beneath the center of capital city
- +10,000 casualties / population 400K
- Massive destruction
- Wave of emigrants
April, 2011: demolition of remaining 1972 wreckage
April, 2014: a new EQ triggers further demolition efforts

Reconstruction?
After 44 years… Not yet

Source: La Prensa, April 20th, 2011

Source: El Universal, April 11th, 2014
How to quantify natural disaster financial impact?

• Simply: we don’t know when an event will occurs!
• But probabilities help us to understand and quantify uncertainty…
• Then considering purpose of measurement!

• Work in progress!!!
1978: Pioneer Seismology Study Applied to Insurance

- Seismic-genetic zones identified ~ Poisson model for EQ frequency
- Attenuation, micro-zonation
- Local structural typology identified
- Detailed exposure database from monopoly state owned insurance company
- Loss functions (% of damage in function of Mercalli)
- Losses calculated in a deterministic way…
- Maximum Possible Loss v. Maximum Probable Loss
Basis of current methodology and risk metrics...

Iso acceleration curves

Average damage v. Intensity Mercalli scale

Average Annual Loss

Country Zonation

Probable Maximum Loss (PML)

- Objective: to quantify the maximum loss amount caused by **ONE** event
- The probability of occurrence /return period directly associated to such event, otherwise deterministic
- Two events were chosen as the ones which may generate highest losses, and the one with the maximum loss is the PML with a recurrence period of $1/x$ years...

Comprehensive Approach for Probabilistic Risk Assessment - CAPRA

Hazard

Exposure

Vulnerability

Results
Hazard: RESIS II: Definition of regional seismic sources

Crustal sources

Interphase subduction sources

Deep Subduction sources

_Amenaza Sísmica en América Central_, Editorial Entinema, 2009
Exposure: Inventory of Exposure ERN-CAPRA-T2-4
Vulnerability (TAP for Water & Sanitation)

\[ P[x \geq ED_{ij}] = f_{ij}(s, \theta_{ij}) \]

\[ ED_{ij} = 0 ; ED_{MAX}(i) = 1 \]

\[ EDM[S] = \sum_{i=1}^{n} P[ED = i] \]

Estado de Daño Medio se deriva de matriz de probabilidad de daño.

\( s \):
vector de parámetros (HAZUS)
Compound distributions for EQ hazard

- Frequency:
  - Number of earthquakes in a given scenario
  - Total number of earthquakes in a year
  - Number of earthquakes by magnitude

- Severity:
  - Conditional loss (given occurrence of an earthquake)
Cat. loss distributions: Why are we simulating?

• If we know the exact loss distribution, then **WE DO NOT NEED TO SIMULATE!**

• In general, we should rely on simulations when dealing with **COMPOUND LOSS DISTRIBUTIONS**
  • Frequency
  • Severity
Simulation Scheme for Seismic Hazard

**Poisson Distributions** \((\lambda_k): N_{k,y}\)
Number of seism associated to scenario “k” (k=1 to m) in year “y” (y=1 to \(n_{sim}\)).

**Beta Distributions** \((\alpha_k, \beta_k \text{ and } \theta_k): X_{k,i,y}\)
Loss associated to “i-th” event (i=1 to \(N_{k,y}\)) in scenario “k” (k=1 to m) in year “y” (y=1 to \(n_{sim}\)).

**How many quakes per scenario?**

**Loss amount from each earthquake in the scenario?**

**Loss amount per scenario (from all earthquakes)?**

**Aggregate annual loss (from all scenarios)?**
Simulation of Compound Distribution
A simple example

\[\begin{align*}
N_1 &= 1 & X_{1,1} &= 12 & S_1 &= 12 \\
N_2 &= 3 & X_{2,1} &= 6; X_{2,2} &= 6; X_{2,3} &= 35 & S_2 &= 47
\end{align*}\]
Simulation of Losses for Costa Rica
Inputs from CAPRA

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Magnitude</th>
<th>Frequency (E[N(i)])</th>
<th>Mean Cond. Loss (E[X(i)])</th>
<th>Var. Cond. Loss (V[X(i)])</th>
<th>Beta Distr. Parameter: α</th>
<th>Beta Distr. Parameter: β</th>
<th>Exposure</th>
</tr>
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<tbody>
<tr>
<td>CAc10_SF1_M=5.380.00323018</td>
<td>5.38</td>
<td>0.00323</td>
<td>73.04</td>
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</tbody>
</table>

Frequency (Poisson: λ)
Conditional Loss Distribution (Exposure, Beta: α and β)
Simulation of Losses in Costa Rica
Worst scenario - but fully probabilistic!

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Magnitude</th>
<th>Frequency: $E[N(i)] = \lambda$</th>
<th>Mean Cond. Loss: $E[X(i)]$</th>
<th>Var. Cond. Loss: $V[X(i)]$</th>
<th>Beta Distr. Parameter: $\alpha$</th>
<th>Beta Distr. Parameter: $\beta$</th>
<th>Exposure</th>
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<tbody>
<tr>
<td>CAsp1_SF45_M=7.280.000121413</td>
<td>7.28</td>
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</tbody>
</table>

Annual Probability of "$n" events

Cumulative Prob. of Severity (Cond. Loss)
Simulation of Losses for Costa Rica
All the scenarios: 1 Year – Summary

<table>
<thead>
<tr>
<th>Number of EQ (Sum of N(i))</th>
<th>Max # of EQs in same scenario</th>
<th>Max Magnitude</th>
<th>Aggregate Annual Loss</th>
<th>Maximum Annual Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1</td>
<td>7.28</td>
<td>2,148.60</td>
<td>1,018.24</td>
</tr>
</tbody>
</table>

We are interested in:
- Aggregate Annual Loss, and/or
- Maximum Annual Loss
## Simulation of Losses for Costa Rica

All the scenarios: 1 Year – Detail of scenarios

We can also analyze losses per scenario:

### CAPRA Output

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Magnitude</th>
<th>Frequency: ( \lambda )</th>
<th>Mean Cond. Loss: ( E[N(i)] \times \lambda )</th>
<th>Var. Cond. Loss: ( V[N(i)] \times \lambda )</th>
<th>Beta Distr. Parameter: ( \alpha )</th>
<th>Beta Distr. Parameter: ( \beta )</th>
<th>Exposure</th>
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<tbody>
<tr>
<td>CAc5_SF21_M=5.290.0245043</td>
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</tbody>
</table>

### Simulation of Number of events, Loss of each event in a scenario and Total losses per Scenario

<table>
<thead>
<tr>
<th>N(i): # EQs at Scenario</th>
<th>Event #1 Loss: X1(i)</th>
<th>Event #2 Loss: X2(i)</th>
<th>Event #3 Loss: X3(i)</th>
<th>Total Loss: S(i)</th>
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</thead>
<tbody>
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<td>1,018.24</td>
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</tbody>
</table>
Simulation of Losses for Costa Rica
All scenarios: 10,000 Years

How many earthquakes per year?

Aggregate annual loss (from all scenarios)?

Are 10,000 years enough?

AAL = 556
Std. Dev. = 1,151
Risk Metrics
For Finance and Insurance
Risk Metrics: Purpose and properties

Some purposes:

- Understanding the risk
- Setting and evaluation of sovereign cat risk strategies
- Development and increase of cat insurance penetration
- Supervision of (re)insurance
- Pricing of risk transfer instruments
- Cost evaluation of financing instruments

Coherent Risk Metrics

Axioms (being A a rd, highest annual loss of a portfolio)

1. Translation Invariance \[ \text{Metric}[A+c] = \text{Metric}[A] +c \]
2. Positive homogeneity \[ \text{Metric}[k*A] = k*\text{Metric}[A] \]
3. Monotonicity (if A>B then \[ \text{Metric}[A] \geq \text{Metric}[B] \])
4. Sub-additivity \[ \text{Metric}[A+B] \leq \text{metric}[A] + \text{metric}[B] \] (diversification benefit)

1 & 2 rules out non-linear algebraic function of event loss. 3) eliminates SD. 4) rules out LaR

Artzner tt al. (1999), cited by G. Woo, BAJ 8, Part V, 2002
Risk Metrics
AAL and Standard Deviation

- Average Annual Loss (AAL)
- Standard Deviation
- Law of Large Numbers
- Commercial premium
- Sound market technical pricing (AAL) while competing on loadings
Metrics for extreme events (looking at the tail)

Deterministic v. full probabilistic approach:

- No longer looking at the worst event, rather looking at a specific loss distribution
- Return period related to physical events occurrence, e.g. earthquakes
- Now annual exceedance probabilities related to financial loss distribution…
Metrics for extreme events (looking at the tail)

- **Loss at Risk (LaR):**
  \[ P[\text{Loss} > \text{LaR}] = \text{Annual Exceedance Probability} \]
  - Minimum amount to be exceed with a given annual probability
  - It is a percentile of the loss distribution
  - It must be reported with its associated annual exceedance probability

- **Expected Tail Loss (ETL):**
  \[ \text{ETL} = \mathbb{E}[\text{Loss} | \text{Loss} > \text{LaR}] \]
  - Conditional expected value over the loss distribution
  - It must be reported with its associated exceedance probability (related to the LaR cutoff)
Risk Metrics of a Loss Distribution

The right border of each bin indicates the cumulative value from zero.
LaR and ETL in a Loss Distribution

LaR 1%

P(Loss > 10.2) = 1%

ETL 1% = 12.8
Average on the tail
Costa Rica: comparing Risk Metrics

Tail Metrics of tail (absolute values)

- CAPRA
- 100,000 years of simulation
Costa Rica: comparing Risk Metrics

Tail Metrics of tail (relative values)

- CAPRA
- 100,000 years of simulation

Loss Distributions

Loss - % of Exposure

Exceedance Probability

0% 4% 8% 12% 16% 20%
0.0% 0.2% 0.4% 0.6% 0.8% 1.0% 1.2% 1.4% 1.6% 1.8% 2.0%

Probability Density

AggregateAnnualLoss
How reliable are the simulation-based Risk Metric estimates?

![Graph showing the reliability of simulation-based Risk Metric estimates.](diagram.png)
Using Risk Metrics
For Risk Financing and Insurance
Pricing of risk transfer product/ evaluation of retention financing funds

- Ratemaking
  - $P = AAL + \alpha \times \text{Std Dev.}$ (usual practice)
  - Other methods (PML-based, etc.)
- Expenses, Target profit, etc.

Conceptual impacts of risk information and pooling on insurance pricing
Layering of Losses (hypothetical example)
Sovereign DRFI strategy/ CXL protection

Obs.: each bin indicates **losses between left and right border**.
Capital Requirement/Catastrophe Reserving

- Ruin theory
- Conservative principle for valuating liabilities
- ETL vs. LaR vs. PML (coherent measure axioms)
- Probabilistic approach: make sense to use aggregate zone sums insured?
- Costa Rica using a return period of 500 years while Perú uses 1,000…
- What about the size of sampling? Standard of 10,000 years of simulation is enough?